SDMC Fermat, November 8, 2008
ANSWERS

1. $\boxed{170°}$ Sum of angles of such a polygon is $(36-2)\times 180° = 6120°$. Therefore, each of 36 angles measures $6120°/36 = 170°$.

2. $\boxed{\text{3 sides}}$ Equilateral triangle is the only regular polygon with acute angles: angles of a square are right; and as the number of sides increases, the angles become obtuse and keep getting bigger.

3. $\boxed{8}$ The four angles of the quadrilateral (part of the polygon not covered with the paper) add up to 360°. If x is the unknown angle of the regular polygon, then $x + x + 39° + 51° = 360$, from which $2x = 270°$ and $x = 135°$ - this is an angle of a regular octahedron.

4. $\boxed{180°}$

5. $\boxed{360°}$ The sum of external angles is 360° for *any* convex polygon.

6. $\boxed{150°}$

7. $\boxed{\text{22 integers}}$ The sum of angle measures of a convex regular polygon with n sides is $(n-2)\times 180°$. Each angle measures:

$$\frac{(n-2)\times 180°}{n} = \frac{180° \times n - 360°}{n} = 180° - \frac{360°}{n}$$

For this number to be integer, n must be a factor of 360. List all factors of 360: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360 (24 factors altogether). Polygons with 1 or 2 angles do not exist, which leaves us with 22 types of regular polygons whose angles' measures are integers.

8. $\boxed{3\sqrt{2}-3}$ Other (non-simplified) forms of this expression are $3(\sqrt{2}-1)$ or $\frac{3}{\sqrt{2}+1}$.

9. $\boxed{\text{48 edges}}$ Use Euler's formula, $V - E + F = 2$. Since $25 - E + 25 = 2$, $E = 48$.

10. $\boxed{\text{Icosahedron}}$ There are twenty triangles in the pattern, so the answer must be obvious for those who know Greek :-)

11. $\boxed{\text{(Tetragonal) pyramid}}$ Other possible descriptions include *rectangular* or *square* pyramid.

12. $\boxed{5}$ This is just the case when $4 - 1 = 5$:-)

13. $\boxed{1/6}$ The solid is a pyramid with triangular (half-square) base having the area of 1/2, and a height of 1.

14. $\boxed{V = 24,\ E = 36,\ F = 14}$ For the cube, $(V, E, F) = (8, 12, 6)$. Each of the 8 vertices, when cut, gives +2 vertices, +3 edges, and +1 face, therefore, for the new truncated solid, $V = 8 + 8 \times 2 = 24$, $E = 12 + 8 \times 3 = 36$, and $F = 6 + 8 \times 1 = 14$. Sanity check: $V - E + F = 24 - 36 + 14$ is still 2.

15. $\boxed{V = 8,\ E = 12,\ F = 6}$

16. $\boxed{V = 7,\ E = 11,\ F = 6}$

17. $\boxed{9/2}$ Whenever the side length of a cube is s, the inscribed octahedron is made of two square pyramids with the base area of $s^2/2$, the height of $s/2$, and thus the volume of

$$\frac{1}{3} \times \frac{s^2}{2} \times \frac{s}{2} = \frac{s^3}{12}$$

The volume of the octahedron is double that, that is $s^3/6$. Plug in $s = 3$ to get $V = 27/6 = 9/2$.

18. $\boxed{2\sqrt{2}}$ The "top view" of the octahedron with the inscribed cube is shown below. In terms of the octahedron edge length, s, the volume of the inscirbed cube is

$$V = \left(\frac{s\sqrt{2}}{3}\right)^3 = \frac{2s^3\sqrt{2}}{27}$$

Plug in $s = 3$ to get $V = 2\sqrt{2}$.

19. $\boxed{\text{5 edges}}$ Entering a new face means crossing an edge, and there are 6 faces. Since the ant is not required to return to the starting point, the last edge may be skipped.

20. $\boxed{\text{4 edges}}$ This time the ant is required to cross the last, 4th, edge.

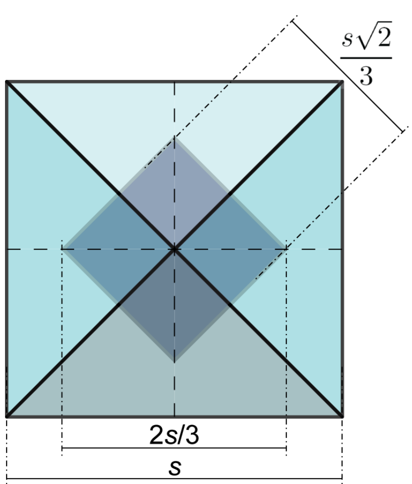
21. $\boxed{\text{7 regions}}$

22. $\boxed{\text{61 regions}}$ Recall the proof of Euler's formula, $V - E + F = 2$. Of the plane, the exterior of the drawing replaces one of the faces, so not counting it means solving $V - E + F = 1$. Therefore, $100 - 160 + F = 1$ and $F = 61$.

23. $\boxed{\text{10 regions with the exterior counted}}$

24. $\boxed{\text{Octahedron}}$

25. $\boxed{\sqrt{2}}$ An octahedron has eight faces, a tetrahedron has four. If the surface areas are the same, then clearly the area of a single face of a tetrahedron must be twice larger than that of the octahedron (and areas relate as squares of side lengths).



Answer to the Bonus Question: $\boxed{4}$ Combined with the answer to #24, it proves that the volume of a regular octahedron is exactly 4 times greater than the volume of a regular tetrahedron with the same edge length.